Study the effects of Earth's gravity on the time delay of radio signals from satellites

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Tenets of General Relativity

- General relativity is the extension of special relativity. It includes the effects of accelerating objects and their mass on spacetime.
- As a result, the theory is an explanation of gravity.
- It is based on two concepts: (1) the principle of equivalence, which is an extension of Einstein’s first postulate of special relativity and (2) the curvature of spacetime due to gravity.

Inertial Mass and Gravitational Mass

- Recall from Newton's 2nd law that an object accelerates in reaction to a force according to its inertial mass:

$$\vec{F} = m_I \vec{a}$$

- Inertial mass measures how strongly an object resists a change in its motion.

$$\vec{F} = m_G \vec{g}$$

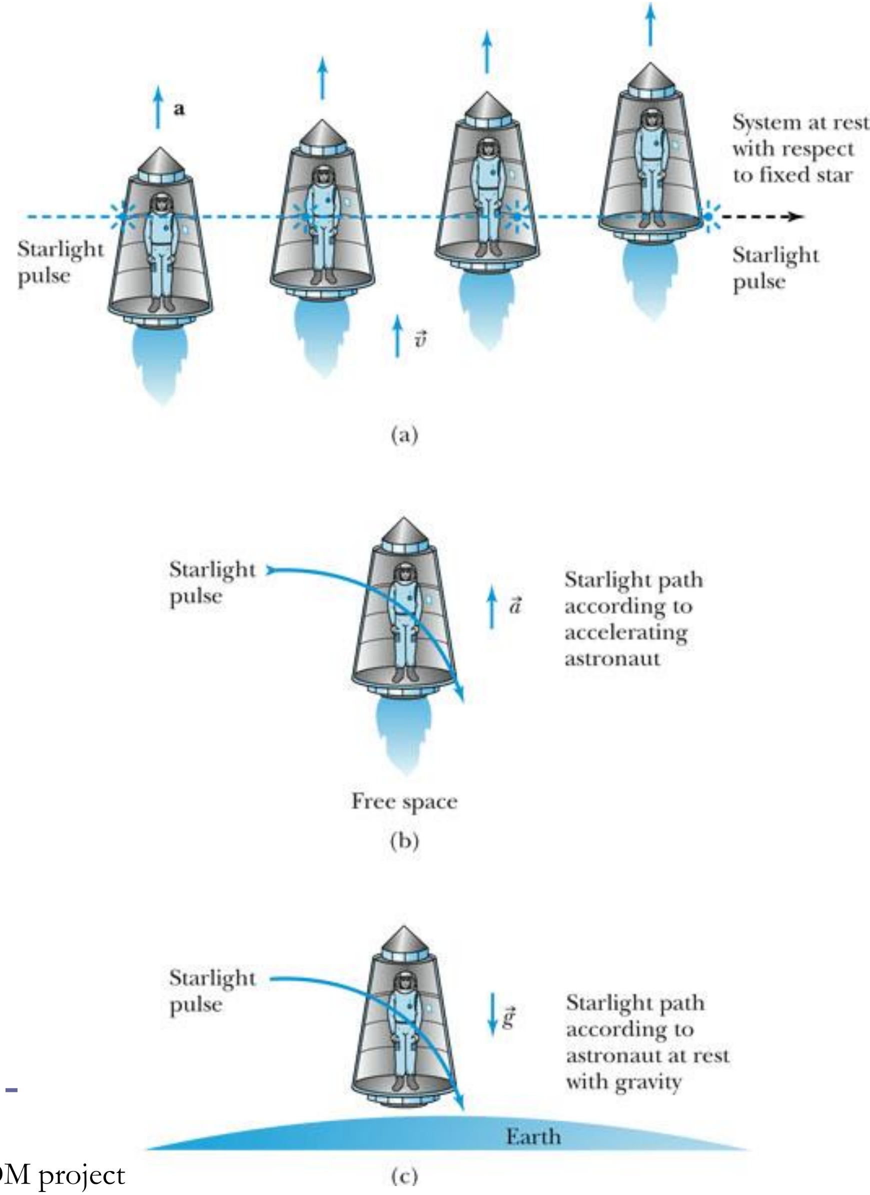
- Gravitational mass measures how strongly it attracts other objects.

- For the same force, we get a ratio of masses: $\vec{a} = \left(\frac{m_G}{m_I} \right) \vec{g}$

- According to the principle of equivalence, the inertial and gravitational masses are equal.

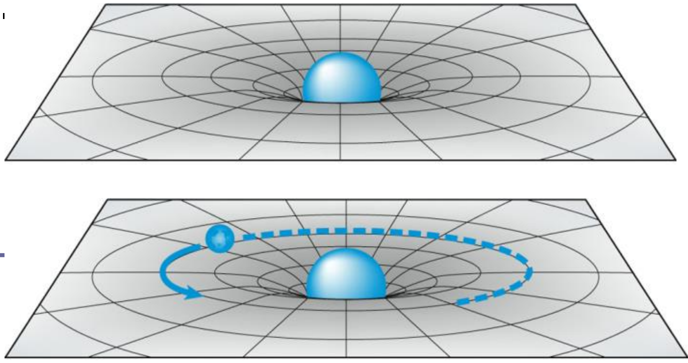
Light Deflection

- Consider accelerating through a region of space where the gravitational force is negligible. A small window on the rocket allows a beam of starlight to enter the spacecraft. Since the velocity of light is finite, there is a nonzero amount of time for the light to shine across the opposite wall of the spaceship.
- During this time, the rocket has accelerated upward. From the point of view of a passenger in the rocket, the light path appears to bend down toward the floor.
- The principle of equivalence implies that an observer on Earth watching light pass through the window of a classroom will agree that the light bends toward the ground.
- This prediction seems surprising, however the unification of mass and energy from the special theory of relativity hints that the gravitational force of the Earth could act on the effective mass of the light beam.



Spacetime Curvature of Space

- Light bending for the Earth observer seems to violate the premise that the velocity of light is constant from special relativity. Light traveling at a constant velocity implies that it travels in a straight line.
- Einstein recognized that we need to expand our definition of a *straight line.*
- The shortest distance between two points on a flat surface appears different than the same distance between points on a sphere. The path on the sphere appears curved. We shall expand our definition of a *straight line* to include any minimized distance between two points.
- Thus if the spacetime near the Earth is not flat, then the straight line path of light near the Earth will appear curved.



The Unification of Mass and Spacetime

- Einstein mandated that the mass of the Earth creates a dimple on the spacetime surface. In other words, the mass changes the geometry of the spacetime.
- The geometry of the spacetime then tells matter how to move.
- Einstein's famous field equations sum up this relationship as:

  * *mass-energy tells spacetime how to curve*

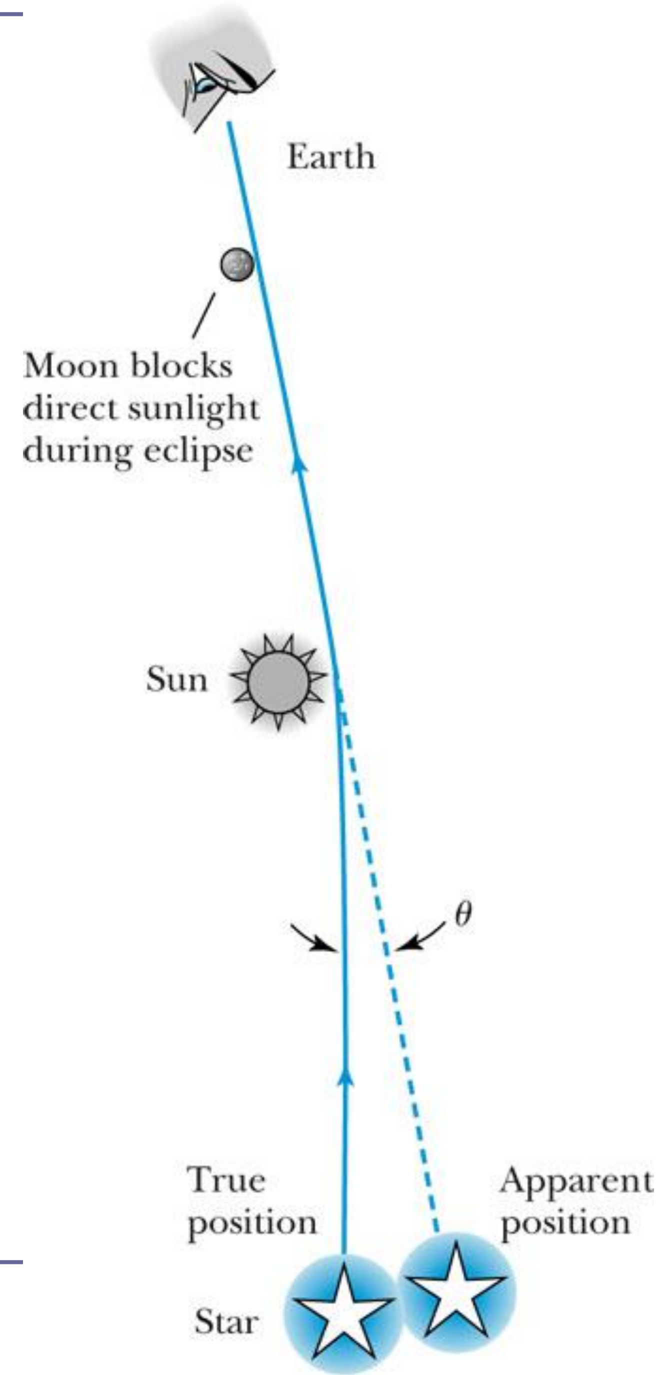
  * *Spacetime curvature tells matter how to move*

- The result is that a standard unit of length such as a meter stick increases in the vicinity of a mass.

Tests of General Relativity

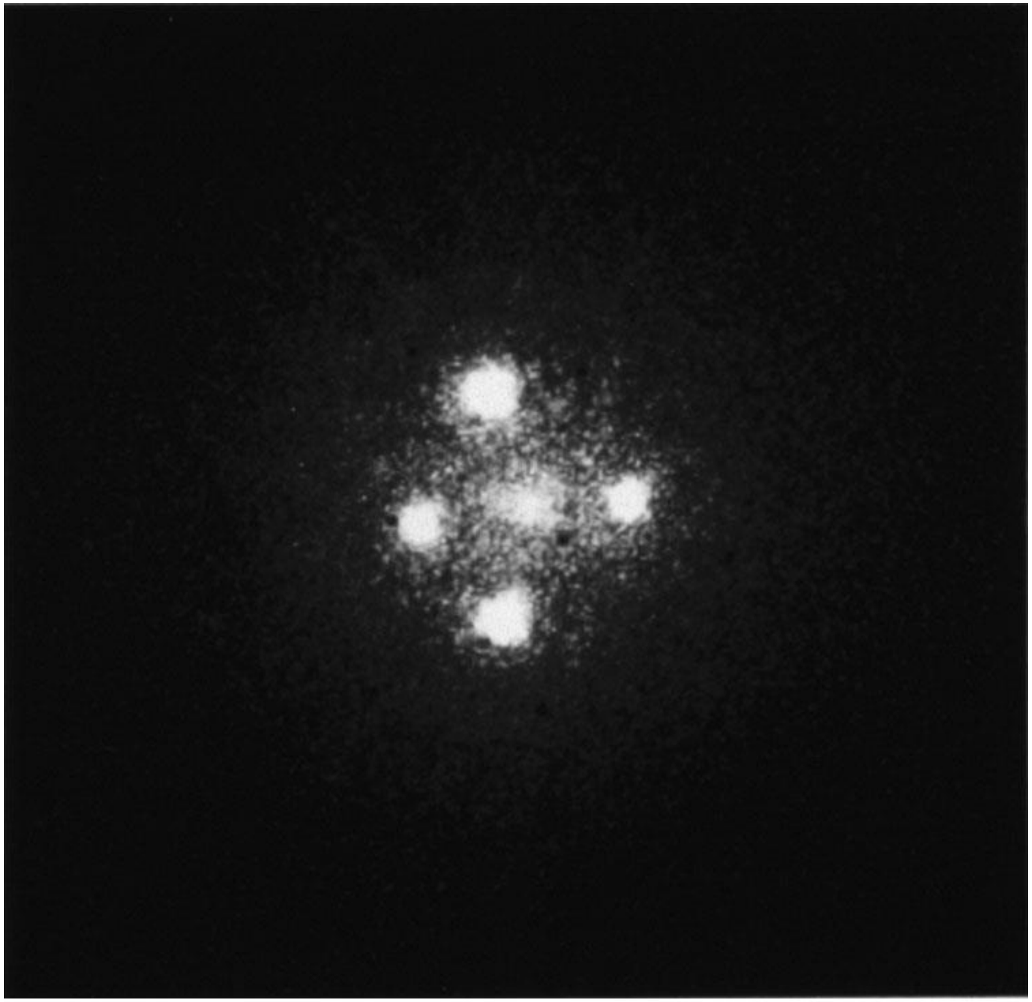
Bending of Light

- During a solar eclipse of the sun by the moon, most of the sun's light is blocked on Earth, which afforded the opportunity to view starlight passing close to the sun in 1919. The starlight was bent as it passed near the sun which caused the star to appear displaced.
- Einstein's general theory predicted a deflection of 1.75 seconds of arc, and the two measurements found 1.98 ± 0.16 and 1.61 ± 0.40 seconds.
- Since the eclipse of 1919, many experiments, using both starlight and radio waves from quasars, have confirmed Einstein's predictions about the bending of light with increasingly good accuracy.



Gravitational Lensing

- When light from a distant object like a quasar passes by a nearby galaxy on its way to us on Earth, the light can be bent multiple times as it passes in different directions around the galaxy.

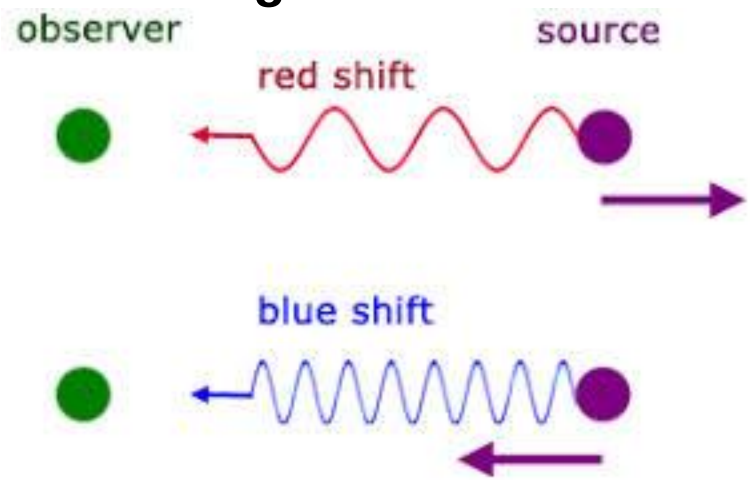


Gravitational Redshift

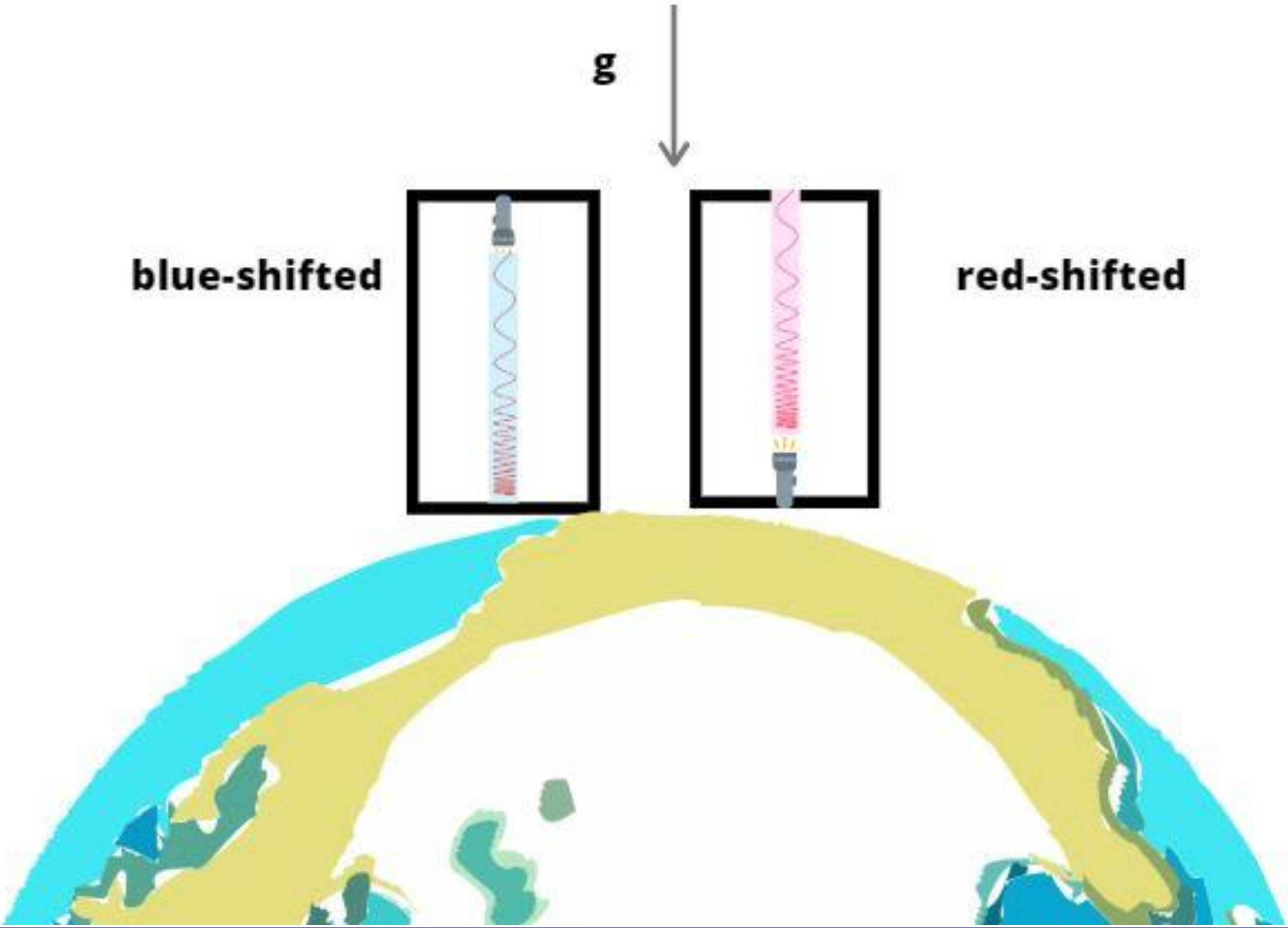
- The second test of general relativity is the

predicted frequency change of light near a massive object.

- A light pulse's energy depends on its frequency *f* through Planck's constant, $E = hf$. As the light pulse travels up vertically, it loses kinetic energy and its frequency decreases. Its wavelength increases, so the wavelengths of visible light are shifted toward the red end of the visible spectrum.
- This phenomenon is called **gravitational redshift**.



Gravitational Redshift



Gravitational Redshift Experiments

- An experiment conducted in a tall tower measured the "blueshift" change in frequency of a light pulse sent down the tower. The energy gained when traveling downward a distance H is mgH. If f is the energy frequency of light at the top and f' is the frequency at the bottom, energy conservation gives $h f = h f' + mgH$.

  The effective mass of light is $m = E / c^2 = h f' / c^2$.

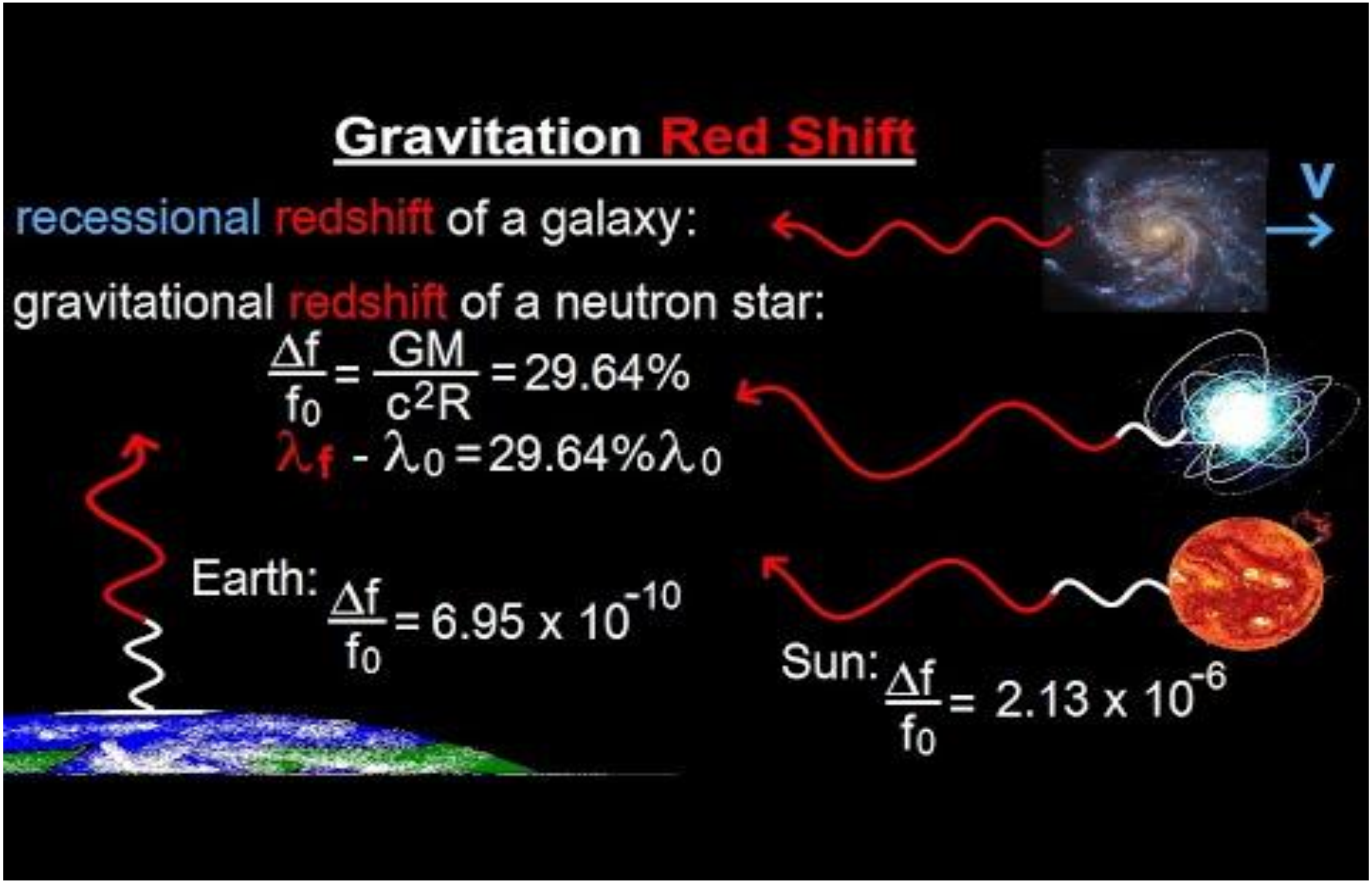
  This yields the ratio of frequency shift to the frequency: $$\frac{\Delta f}{f} = \frac{gH}{c^2}$$

  Or in general:

$$\frac{\Delta f}{f} = -\frac{GM}{c^2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

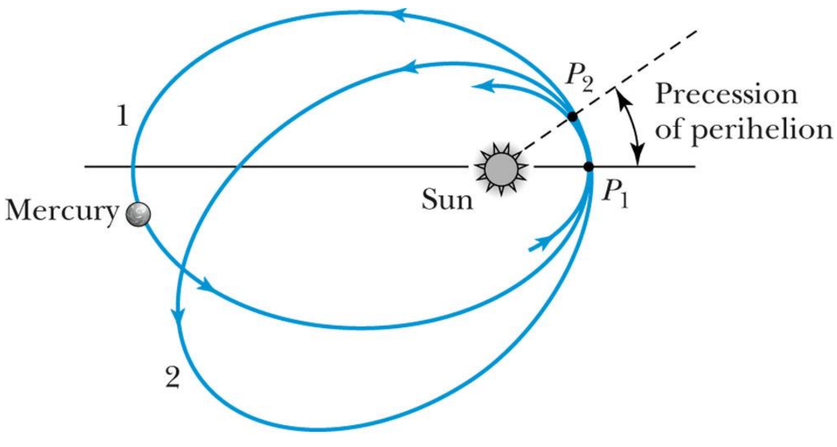
  Using gamma rays, the frequency ratio was observed to be:

$$\Delta f / f \approx 10^{-15}$$


Gravitation Red Shift
V
recessional redshift of a galaxy:
gravitational redshift of a neutron star:
$\frac{\Delta f}{f_0} = \frac{GM}{c^2R} = 29.64\%$
$\lambda_f - \lambda_0 = 29.64\% \lambda_0$
Earth: $\frac{\Delta f}{f_0} = 6.95 \times 10^{-10}$
Sun: $\frac{\Delta f}{f_0} = 2.13 \times 10^{-6}$

Perihelion Shift of Mercury

- The orbits of the planets are ellipses, and the point closest to the sun in a planetary orbit is called the perihelion. It has been known for hundreds of years that Mercury's orbit precesses about the sun. Accounting for the perturbations of the other planets left 43 seconds of arc per century that was previously unexplained by classical physics.
- The curvature of spacetime explained by general relativity accounted for the 43 seconds of arc shift in the orbit of Mercury.

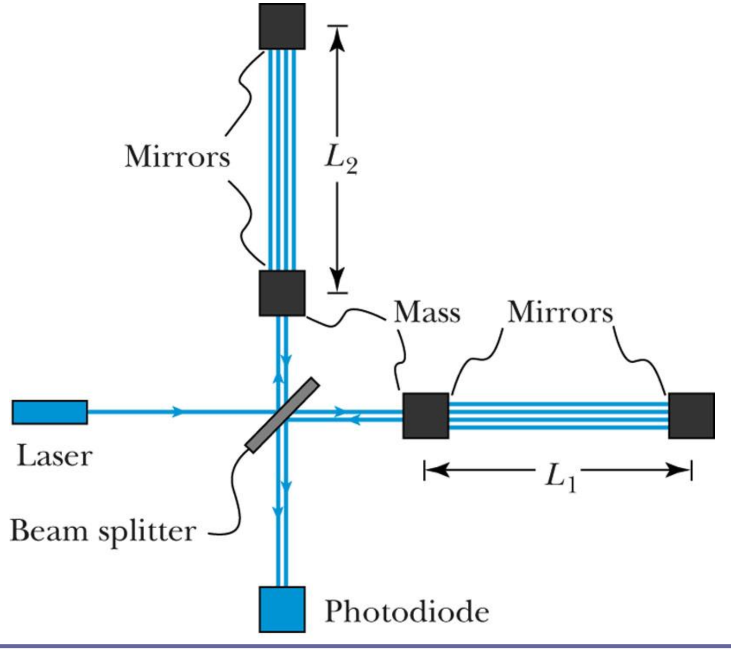


Gravitational Waves

- When a charge accelerates, the electric field surrounding the charge redistributes itself. This change in the electric field produces an electromagnetic wave, which is easily detected. In much the same way, an accelerated mass should also produce gravitational waves.
- Gravitational waves carry energy and momentum, travel at the speed of light, and are characterized by frequency and wavelength.
- As gravitational waves pass through spacetime, they cause small ripples. The stretching and shrinking is on the order of 1 part in 10^{21} even due to a strong gravitational wave source.
- Due to their small magnitude, gravitational waves would be difficult to detect. Large astronomical events could create measurable spacetime waves such as the collapse of a neutron star, a black hole or the Big Bang.
- This effect has been likened to noticing a single grain of sand added to all the beaches of Long Island, New York.

Gravitational Wave Experiments

- Taylor and Hulse discovered a binary system of two neutron stars that lose energy due to gravitational waves that agrees with the predictions of general relativity.
- LIGO is a large Michelson interferometer device that uses four test masses on two arms of the interferometer. The device will detect changes in length of the arms due to a passing wave.

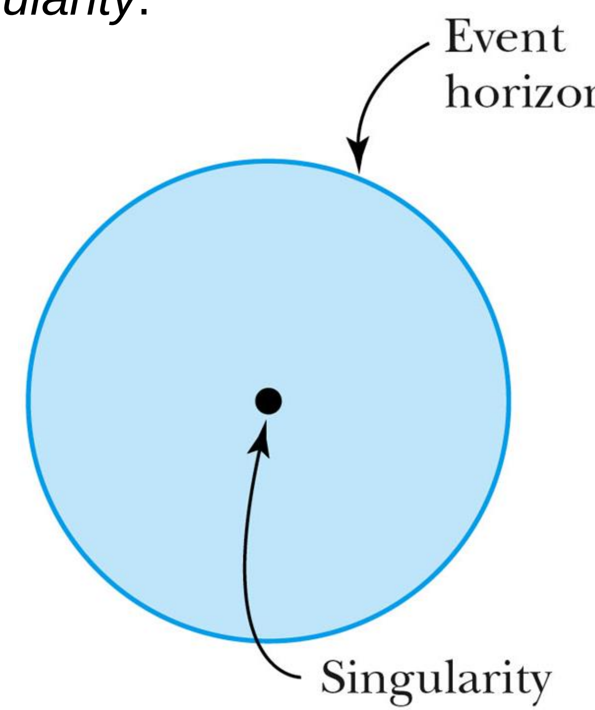


- NASA and the European Space Agency (ESA) are jointly developing a space-based probe called the Laser Interferometer Space Antenna (LISA) which will measure fluctuations in its triangular shape.

Black Holes

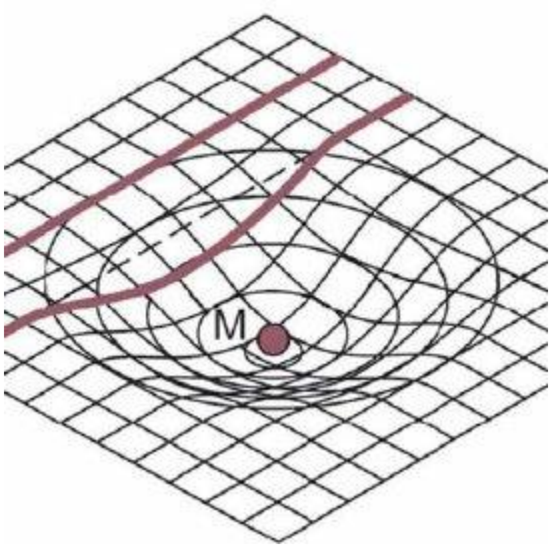
- While a star is burning, the heat produced by the thermonuclear reactions pushes out the star's matter and balances the force of gravity. When the star's fuel is depleted, no heat is left to counteract the force of gravity, which becomes dominant. The star's mass collapses into an incredibly dense ball that could wrap spacetime enough to not allow light to escape. The point at the center is called a *singularity*.
- A collapsing star greater than 3 solar masses will distort spacetime in this way to create a black hole.
- Karl Schwarzschild determined the radius of a black hole known as the *event horizon*.

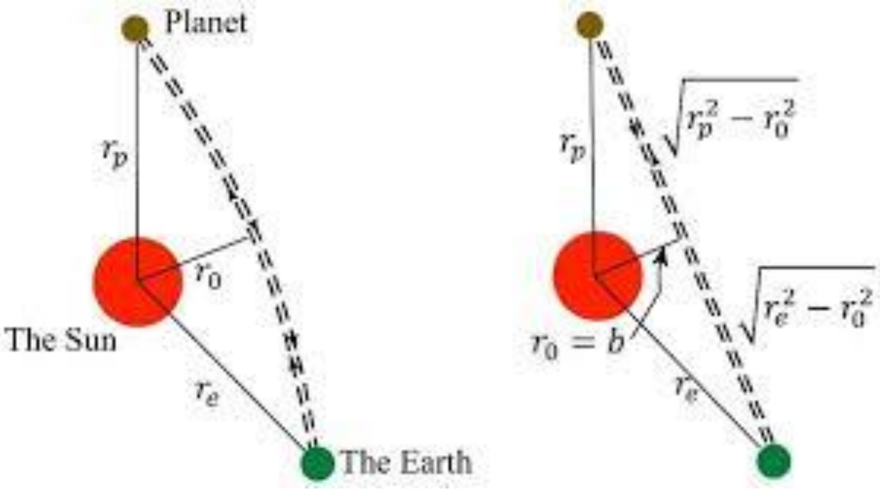
$$r_S = \frac{2GM}{c^2}$$



Gravitational time delay

- Difference of travel time of EMW between A and B through curved and straight lines.

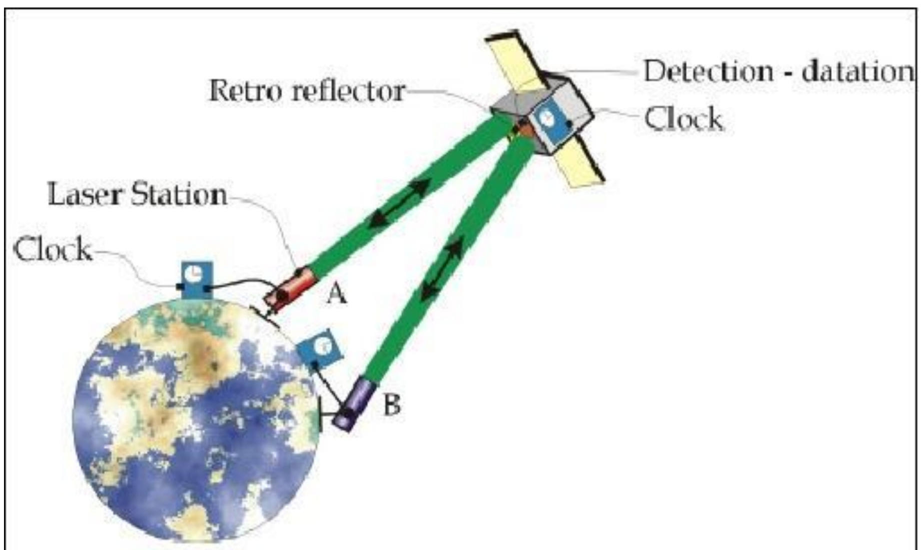


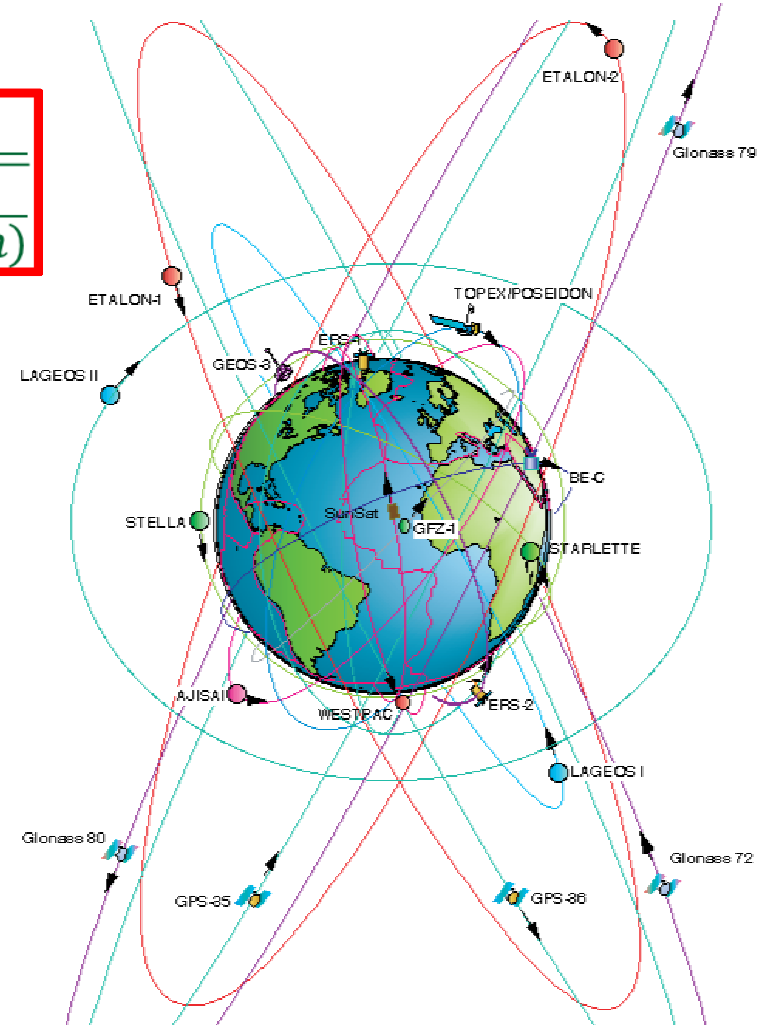


Time measurements

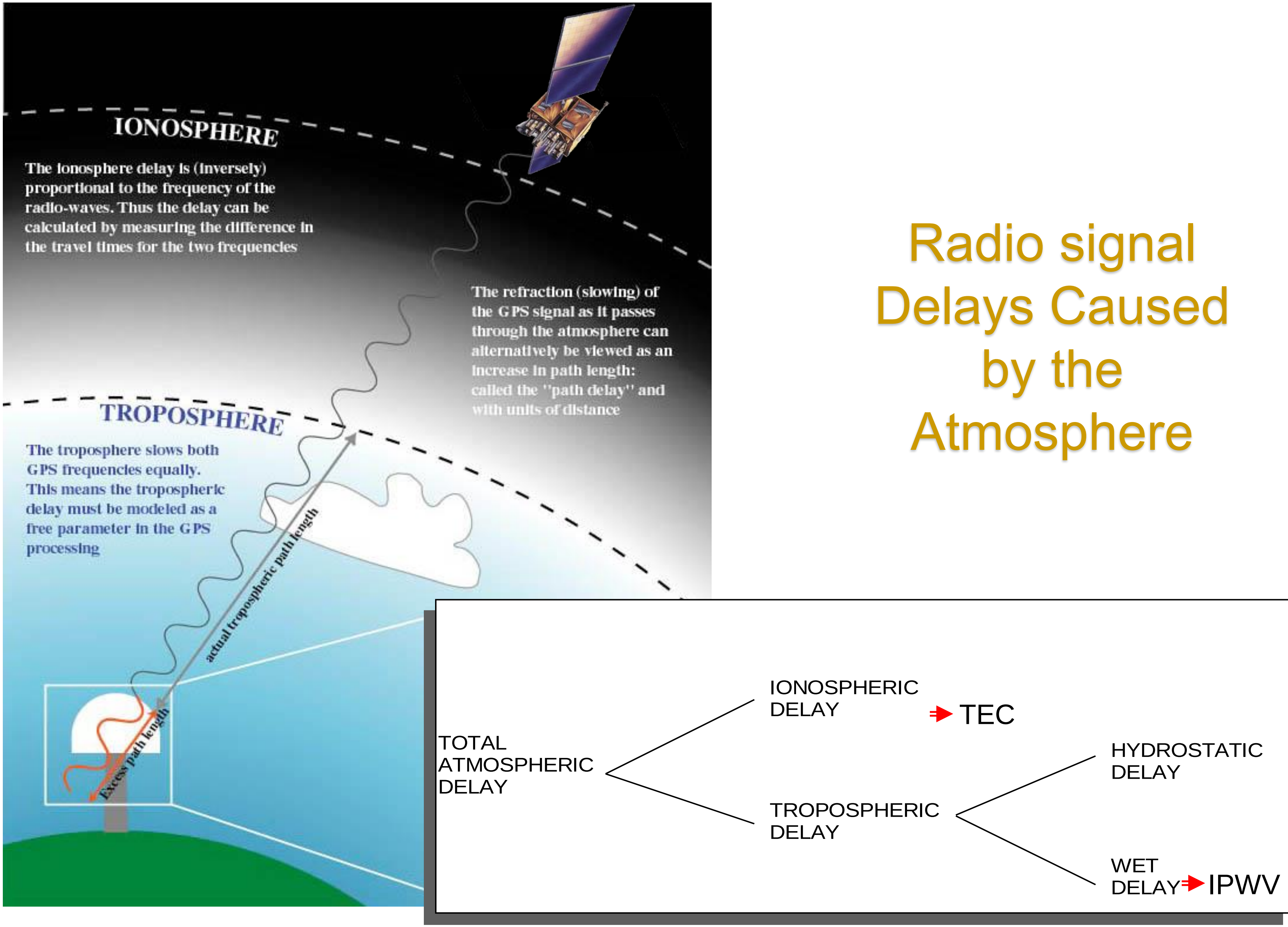
- Proper time $$\Delta t = \frac{\Delta\tau}{\sqrt{1 - \frac{2GM}{c^2(R+h)}}}$$

Clock slows down near Massive object



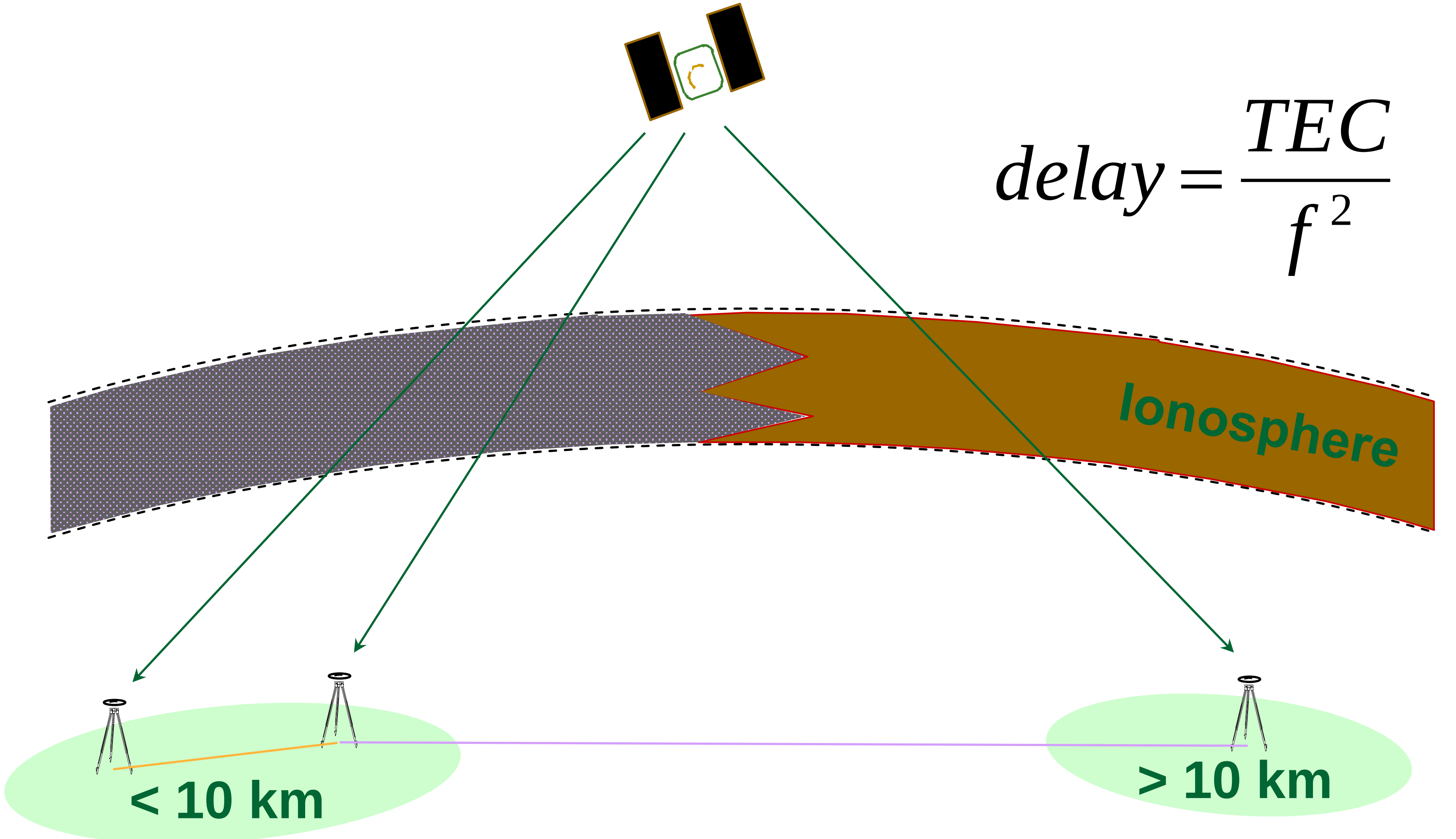


Radio signal Delays Caused by the Atmosphere



Atmosphere based Ionospheric Delay

$$delay = \frac{TEC}{f^2}$$

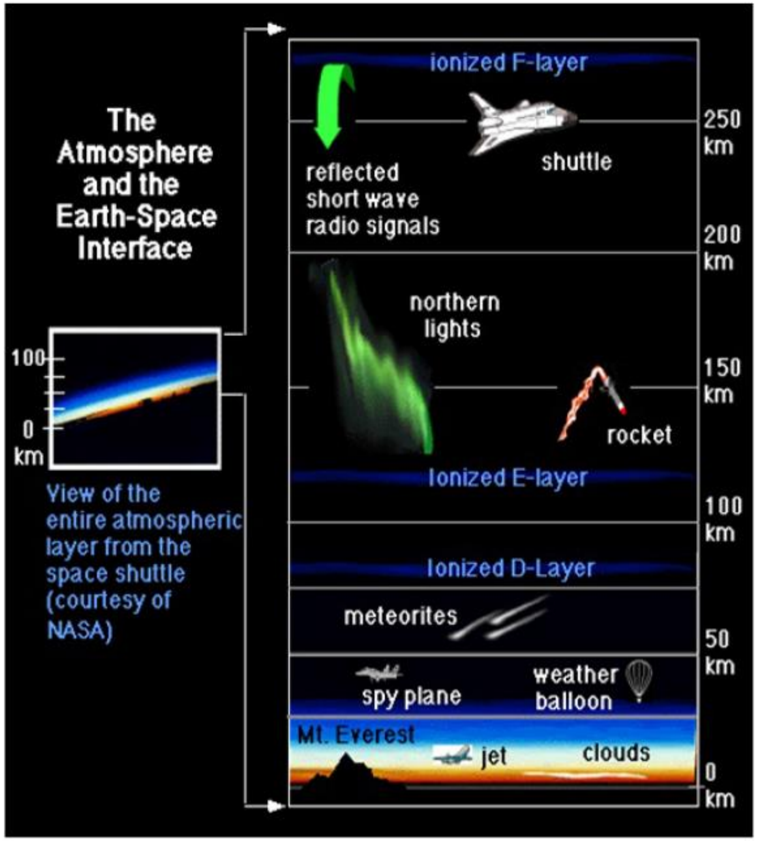


- The ionosphere is
  - A region of earth's atmosphere where uv and x-ray radiation from sun cause gas ionization.
  - Extends from 50 km to ~1,000 km altitude
- Ionosphere is a dispersive medium – it bends GPS signals and changes propagation speed of signal.
  - Bending (signal refraction) causes negligible range errors
  - Propagation speed changes cause significant range errors
    - Speeds up carrier phase beyond speed of light, so ranges appear short
    - Slows down PRN code, so ranges appear long
- Ionosphere is not homogenous – described in layers within which electron densities vary.
  - Total Electron Count (TEC) varies with time of day, time of year, 11-year solar cycle, geographic location.

Ionosphere is mainly divided into D, E, and F regions

IONOSPHERE

- D-layer –(60 - 90 km)
  $n \sim 10^2 - 10^3\ cm^{-3}$
- E-layer – (90 - 140 km)
  $n \sim 10^5\ cm^{-3}$
- F-layer – (140– 1000 km)
  $n \sim 10^5 - 10^6\ cm^{-3}$



Ionosphric studies in AI

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Relativistic Astrophysics General Relativity and Grav... Neutron Stars

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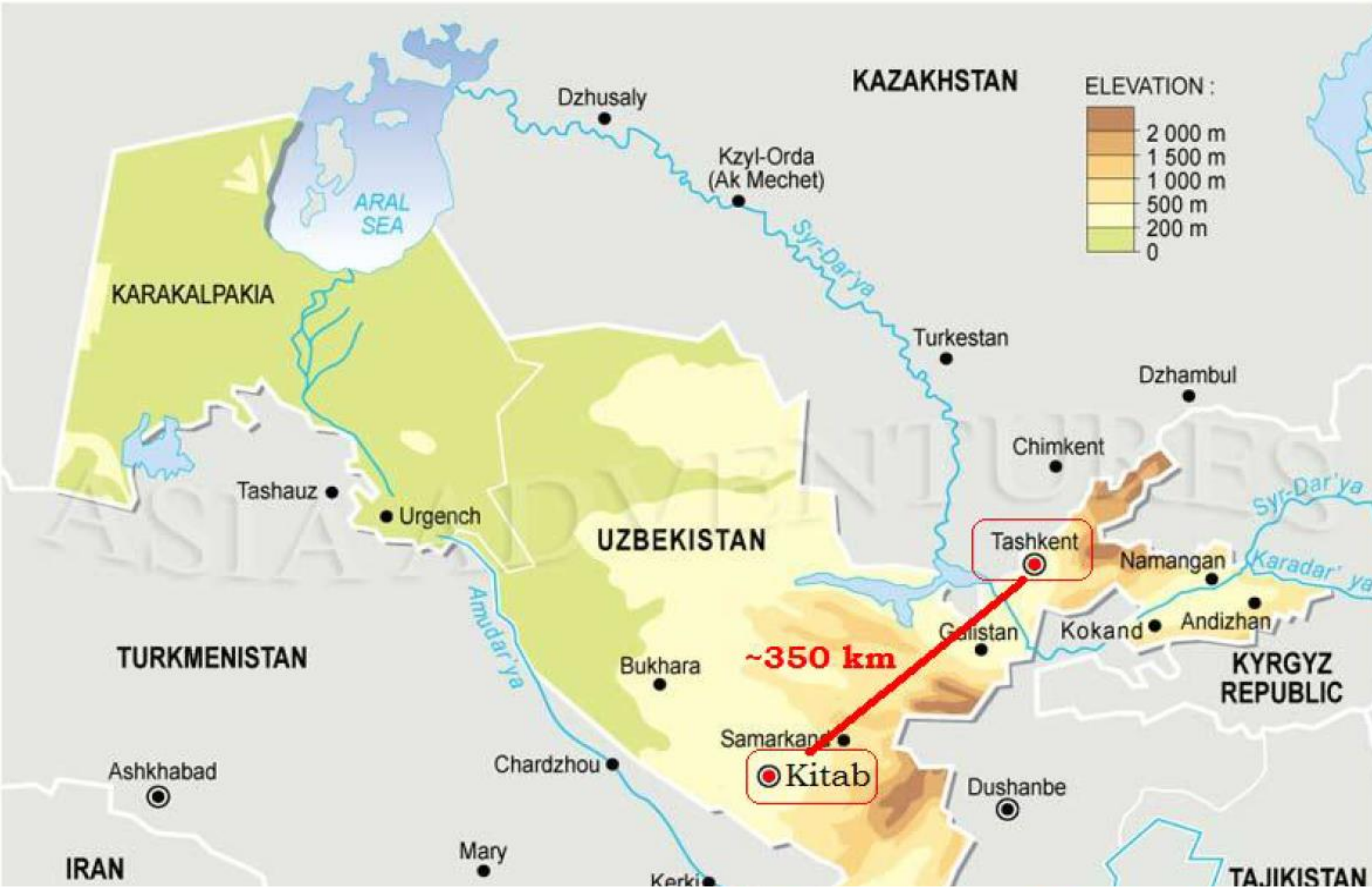
General Relativity and Grav... Relativistic Astrophysics Neutron Stars Black holes Ionosphere

GNSS permanent stations in Uzbekistan



GPS Station in Tashkent and Kitab

Maidanak complex of telescopes

TEC extraction

GPS measurements use time delay between radio signals at two frequencies.

$$L_1 = 1575.42\text{MHz} \quad \text{and} \quad L_2 = 1227.6\text{MHz}$$

Pseudorange.

$$P_i = \rho + c\left(dt_{rec} - dt^{sat}\right) + \Delta_i^{iono} + \Delta^{trop} + Rel_{rec}^{sat} + K_{rec} + K^{sat}$$

Effect of F-layer of ionosphere with thickness 870 km.

$$\Delta_i^{iono} = \frac{40.3}{L_i^2} \int_{z_{rec}}^{z_{sat}} N(z)dz = \frac{40.3}{L_i^2}\mathbf{TEC}$$

Total Electron Content (TEC).

$$\mathbf{TEC} = \frac{L_1^2 \cdot L_2^2}{40.3\left(L_1^2 - L_2^2\right)}\left(P_1 - P_2\right)$$

Receiver Independent EXchange (RINEX) FORMAT

observation file
kit31450a.13o

navigation file
kit31450a.13n

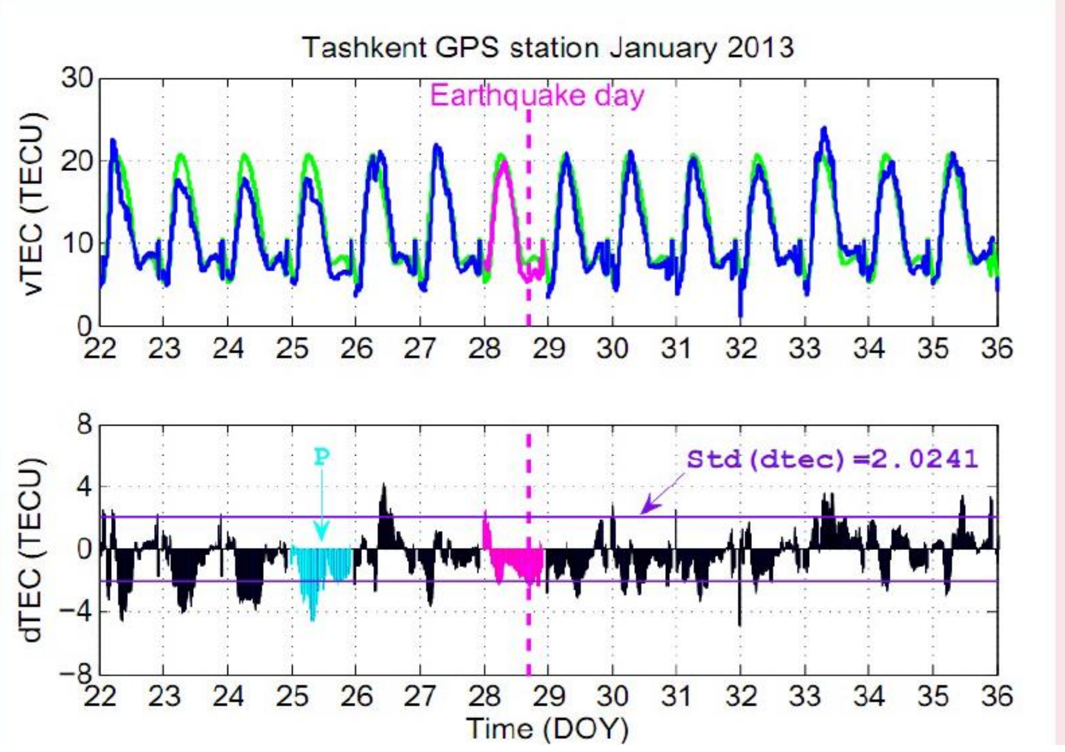
Kitab GPS station

ftp://cddis.gsfc.nasa.gov/gps/data/daily/2013/145/00/kit31450a.13o
ftp://cddis.gsfc.nasa.gov/gps/data/daily/2013/145/00/kit31450a.13n

- Pseudo lengths P1 and P2 are extracted from observation file
- Ephemerides (coordinates & orbits of satellites) are extracted from navigation file

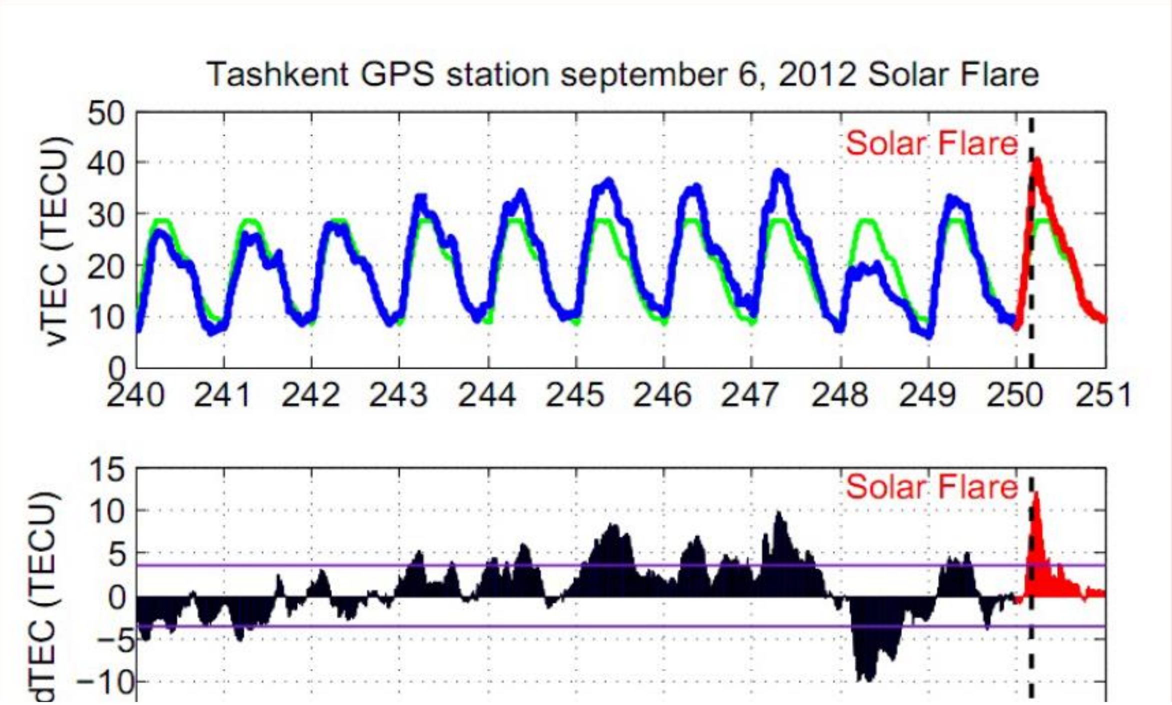
Tashkent GPS station: January 28, 2013

Vertical and Differential TEC variations above Tashkent for 14 consecutive days, including KAZAKHSTAN earthquake (M=6.1) date: January 28, 2013 (day 28) in comparison with the monthly mean (green line), P character denote the precursor day.



Tashkent GPS station: September 6, 2012, Solar Flare

Vertical and Differential TEC variations above Tashkent for 11 consecutive days, including Solar Flare date: September 6, 2012 (day 250) in comparison with the monthly mean (green line). vTEC value on the day the flash (indicated by the red) shows an increase of up to 40 TECU.



Thank you for your attention